- Integration is the backwards of doing a derivative - **antidifferentiation**
- An integral represents the signed **area under the curve** of $f(x)$
  - If the curve is below the x-axis, then the integral is negative the area.
- **Definite** integrals - has bounds, is a definite value.
- **Indefinite** integrals - no bounds, is a function (don't forget to add C!).
- Power rule: $\int x^n dx = \frac{x^{n+1}}{n+1} + C$
  - By extension: $\int cx^n dx = \frac{cx^{n+1}}{n+1} + C \qquad n \neq -1$
- **Slope field**: graphs the slope at a point using a differential equation
- If $f(x)$ is continuous, it is integrable!
- **Area between two curves** = definite integral of the upper curve minus the lower curve.
  - Area between $f(x)$ and $g(x)$ on the interval [a, b] given that $f(x) \geq g(x)$ on [a, b] is $\int_a^b f(x) - g(x)dx$
- Properties of integrals:
  - $\int_a^a f(x)dx = 0$ $\qquad$ $\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$
- **Fundamental Theorem of Calculus**: $\int_a^b f'(x)dx = f(b) - f(a)$.
  - This means that the definite integral is the change of the antiderivative on an interval.
- **Second Fundamental Theorem of Calculus**: $\frac{d}{dx}\int_a^x f(t)dt = f(x)$
  - Extension by chain rule: $\frac{d}{dx}\int_a^{g(x)} f(t)dt = f(g(x))g'(x)$
  - If your lower bound is not a constant, then rewrite the integral such that it is so.
    - $\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)dt = \frac{d}{dx}\int_a^{h(x)} f(t)dt - \frac{d}{dx}\int_a^{g(x)} f(t)dt$
- **Mean Value Theorem of Integrals**: If a function $f(x)$ is continuous on [a, b] where a < b and differentiable on (a, b), then there exists a c where $c \in (a, b)$ such that $\int_a^b f(x)dx = f(c)(b-a)$.
  - Extension: Average value on an interval [a, b]: $\frac{\int_a^b f(x)dx}{b-a}$